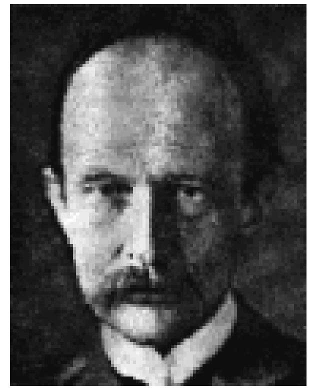
Final Project: The Planck Function

Exploring the properties of blackbody radiation



Max Planck 1858–1947

Background

Any physical body with a finite temperature emits thermal radiation. You can often see this radiation (sunlight, the glow of a fire) or feel it as 'heat' (you are sensing infra-red radiation). A blackbody is an idealized body that completely absorbs all thermal radiation incident upon it, and emits thermal radiation with a very specific spectrum (brightness as a function of wavelength). Remarkably, the form of the spectrum cannot be fully understood in terms of classical physics (google "ultraviolet catastrophe"). It wasn't until the great German physicist Max Planck introduced the idea that energy can be made up of small "packets" or "quanta", that blackbody radiation was finally understood. The result was the famous Planck function,

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda k_B T} - 1},$$

where $B_\lambda(T)$ is the energy (in Joules) emitted per unit wavelength per second per square meter of projected surface area per steradian of solid angle for a perfect blackbody at temperature T. The remaining symbols are wavelength, λ, the speed of light, c, Boltzmann's constant, k_B, and Planck's constant, h. Planck's work on blackbody radiation, for which he received the Nobel Prize in 1918, laid the foundation of quantum mechanics.

The program

Design, write, and test one or more PYTHON functions to perform the following tasks:

- Plot the blackbody spectrum for a given temperature, to be set by the user.
- Locate the wavelength at which the spectrum peaks for a given temperature.
- Compute the intensity I, i.e., the total energy emitted per second per square meter by a blackbody, at a given temperature. This will involve integrating the Planck function with respect to both solid angle (over a hemisphere) and wavelength:

  $$I = \int_0^\infty d\lambda \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \, B_\lambda(T)\cos(\theta)\sin(\theta)$$

  Here, $d\Omega = \sin(\theta)\, d\theta\, d\phi$ is the infinitesimal solid angle on a hemisphere and the $\cos(\theta)$ term accounts for the fact that you are looking at the so-called "projected area" of the surface. Since $B_\lambda(T)$ is independent of angle, the integral over the angles θ and ϕ can easily be done analytically, leaving just the integral over wavelength to be done numerically.

Programming issues:

- The Planck function is not bounded in wavelength but in your program you will need to invent a sensible way to set lower and upper wavelength limits. Since the wavelength at which the spectrum peaks is inversely proportional to the temperature of the blackbody (Wien's Law), these limits cannot simply be set at fixed wavelengths.
- It is also useful to rewrite your integral in terms of a dimensionless variable. In fact, this may help you to deal in part with the previous programming issue.

Investigation

Use your program(s) to explore various properties of blackbody radiation. Here are some suggestions:

- Determine the wavelengths at which the following entities emit thermal radiation most strongly: Cosmic Background Radiation; the Sun; the Earth; Mercury; yourself. In which parts of the electromagnetic spectrum does the radiation peak in each case? Estimate the total amount of radiation that you emit yourself.
- The stars Betelgeuse and Rigel are in the constellation of Orion. Even to the naked eye, they have different colors (check this yourself if the night is clear and you can find a suitably dark place). Why?
- What is fraction of the total radiated power that is emitted in the visible spectrum by various types of star (e.g., the Sun, Betelgeuse etc.)?
- Confirm that the wavelength at which radiation emitted by a blackbody peaks is inversely proportional to its temperature (Wien's displacement law).
- Confirm that for a blackbody, the total energy emitted per square meter of its surface per second is proportional to the 4^{th} power of its temperature (Stefan-Boltzmann Law).
- Broadly speaking, the Earth's average surface temperature is determined by the balance between the rate at which energy is absorbed from the sun and the rate at which energy is radiated into space by the Earth. Develop a simple model for the Earth's equilibrium temperature assuming that its surface radiates like a blackbody. What is the characteristic wavelength of the radiation emitted by the Earth? Consider how the Earth's atmosphere might modify the simple picture. Are there constituents of the atmosphere that absorb strongly in some spectral ranges but not others and what consequences does this have?

Resources

http://hyperphysics.phy-astr.gsu.edu/hbase/mod6.html (introduction to blackbody radiation)

http://astro.unl.edu/naap/ (Various applet based tutorials illustrating concepts in astronomy, including blackbody radiation.)

http://scienceworld.wolfram.com/physics/PlanckLaw.html (a more mathematical description)

http://www.nobelprize.org/nobel_prizes/physics/laureates/1918/planck-bio.html (A short biography of Max Planck.)