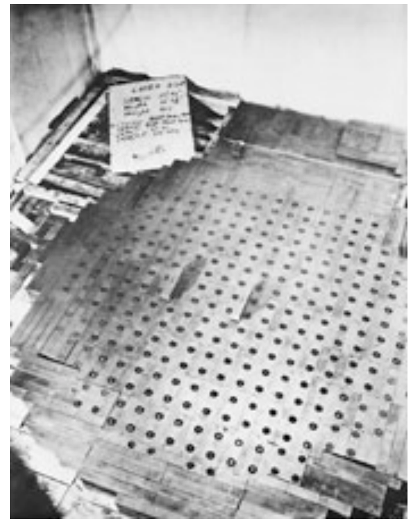
Final Project: Critical Mass

Neutron diffusion in U^{235}

Background



The “atomic pile” in which Enrico Fermi and Leo Szilard creating the first self-sustaining neutron chain reaction.

The 1930’s saw rapid advances in nuclear physics, leading in 1939 to the experimental discovery of nuclear fission by German chemists Hahn and Strassmann and its correct interpretation by refugee Austrians Lise Meitner and Otto Frisch. In this process, the nucleus of a heavy element such as Uranium 235 breaks apart into nuclei of lighter isotopes following a collision with a neutron. The combined mass of the fragments is smaller than the mass of the original nucleus and an amount of energy $E = \Delta mc^2$, equivalent to the difference in mass, is released. The amount released in the fission of a single nucleus is tiny, but each fission event also produces several more neutrons, which can in turn collide with further U^{235} nuclei. It was quickly realized that, in principle, a runaway chain reaction could release vast amounts of energy, making it possible to create a devastating weapon — the so-called atomic bomb. In August 1939, concerned that Nazi Germany might be developing such a weapon, Albert Einstein sent a letter (actually drafted by the brilliant Hungarian-born physicist Leo Szilard) to President Franklin D. Roosevelt urging that the USA begin its own program of research. A few weeks later, the German army invaded Poland, beginning the Second World War.

“Einstein’s letter” eventually led to the Manhattan Project, a concentrated effort by the Allies to develop the atomic bomb before the Nazis.[1] As is well known, the Manhattan Project was ultimately successful, but along the way many fundamental problems had to be solved and many technical difficulties overcome. The key physics problem was that any practical device has surfaces through which neutrons will escape. So, under what conditions can a chain reaction be sustained? This boils down to the question what is the *critical mass* of fissionable material needed so that the chain reaction becomes self-sustaining. This is the question you will investigate.

This problem can be treated as a diffusion problem. If the number density (i.e. number of neutrons per m^3) of neutrons at a particular position $\vec{r}$ at time t is $n(\vec{r},t)$ then the rate of change of n with time at this point is given by

$$\frac{\partial n}{\partial t} = D\nabla^2 n + Cn$$

[1] Ironically, it became clear after the war that the Nazis never got anywhere near to a practical weapon.

where D is the diffusion coefficient, C is rate at which neutrons are created (in the fission process). Note that ∇ is the gradient operator, so the term $\nabla^2 n$ expands as

$$\nabla^2 n(\vec{r},t) = \frac{\partial^2 n(x,t)}{\partial x^2} + \frac{\partial^2 n(y,t)}{\partial y^2} + \frac{\partial^2 n(z,t)}{\partial z^2}.$$

Things become more familiar if we consider only one dimension. Then the equation becomes

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2} + Cn$$

with $n = n(x,t)$. This is the same as the heat conduction equation encountered in Project 6, except for the additional term Cn, which accounts for the creation of neutrons. This equation can be solved using a similar finite difference scheme using appropriate initial values and boundary conditions.

Modeling multi-dimensional diffusion requires methods that are beyond the scope of the course, so you should probably attack only the 1-dimensional case.

The program

Design, write and test a PYTHON program to compute and plot the neutron density $n(x,t)$ both as a function of position (x) along a thin rod of U^{235} of length L and as a function of time (t). For U^{235}, you can assume that $D \approx 10^5$ m^2/s and $C \approx 10^8$ s^{-1}.

The first thing you will need to do is rewrite the partial differential equation as a finite difference equation. You are advised to show your equations to your instructor before you start work on the program itself.

A good starting point is the `hotrod` program of Project 6. Since we are dealing with a rod of constant cross-section and density, its length L establishes the critical mass. This will be an input parameter. You might also want to make C and D input parameters so that different values can be explored.

Your program should

- Accept different sets of boundary and initial conditions.
- Compute the time evolution of the neutron density $n(x,t)$ along the rod up to some user-specified end time.
- Produce a "wire mesh" plot of $n(x,t)$.
- Compute the average neutron density for the rod at each time-step (i.e. $\langle n(t) \rangle$) and plot this quantity versus time.

Programming issues: To produce a wire mesh plot you will need to use a 2-d array to record the neutron density in each element of the rod at each timestep. Remember that the explicit finite difference method is prone to numerical instabilities; you should include a check that the input parameters do not violate the stability criterion.

Investigation

The objective is to determine the critical mass (i.e., the length L) for which a neutron chain reaction is self-sustaining. You should try to use dimensional analysis to get a rough estimate of what the critical value of L will be. That is, can you form a quantity having the dimensions of length from the parameters given in the problem?

For your numerical investigations, the first thing to consider is: What kind of behavior would you expect $n(x,t)$ to exhibit for sub-critical and super-critical systems, respectively? You should be careful to distinguish super-critical behavior from numerical instabilities, which tend to produce exponential growth with short-period or -wavelength oscillations.

Clearly, for each set of initial conditions, the key parameter to vary is L. Here are some suggestions for investigation.

Set up the rod with boundary conditions n(left) = n(right) = 0 (here we are assuming that, somehow, neutrons only leak out of the ends of the rod). Suppose a pulse of neutrons is injected at the center of the rod, so the initial values are $n(x,0)=0$ zero everywhere except at the center, where $n(0,0) = n_0$ (taking the origin of x to be the center of the rod). Try several different combinations of L and n_0. Can you trigger a chain reaction? Does the value of n_0 have any effect on whether the system is sub-critical (no sustained chain reaction) or super-critical (self-sustaining chain reaction)? What is the behavior of $\langle n(t) \rangle$ in these two cases? Can you model this behavior with a simple mathematical function? Is there a characteristic time associated with the behavior?

Actually U^{235} is naturally fissionable, i.e., there is a small probability that it will undergo fission spontaneously, so providing its own neutrons (which is what makes a bomb practical). To simulate this, set all the rod elements to the same initial value n_0. How does this affect the subsequent behavior of $n(x,t)$ and $\langle n(t) \rangle$?

In practice, the Uranium was surrounded by an inert material (known as a "tamper"), in which neutrons still diffuse but are not generated. In our case, the ends of the rod would be inert material with the Uranium sandwiched in the middle. To simulate this, set C=0 for $|x| > a$, where $a < L/2$ is the distance along the rod, measured from the center, at which the Uranium/inert material boundary is placed. Why did they do this? What happens to the critical mass as the fraction of the rod made of inert material is increased?

What if neutrons were continuously injected at one end of the rod? Explore this case by making one of the boundary conditions non-zero.

Resources

Tipler, P. & Llewellyn, R., Modern Physics, Freeman. Or any equivalent modern physics textbook.

http://hypertextbook.com/eworld/einstein.shtml, Einstein's original letter to Roosevelt.

http://www.dannen.com/szilard.html, a biographical web site devoted to Leo Szilard, who was the first to realize the possibility of a neutron chain reaction (prior to its experimental discovery) and patented the idea in an effort to keep it secret and so prevent its use. Horrified and disillusioned by the post-war nuclear arms race he became active in the peace movement and abandoned physics for biology.

http://library.thinkquest.org/17940/texts/timeline/manhattan.html for background on the Manhattan Project.

http://www.lanl.gov/history/index.shtml Los Alamos National Labs, where the Bomb was built.

http://hyperphysics.phy-astr.gsu.edu/hbase/nuccon.html#nuccon An introduction to nuclear physics.

http://www.fas.org/sgp/othergov/doe/lanl/docs1/00349710.pdf the original Los Alamos Primer; a set of notes based on a series of introductory lectures explaining the basic theory of the Bomb. These were handed out to scientists joining the project.

http://blogs.scientificamerican.com/observations/2011/07/18/nuclear-fission-confirmed-as-source-of-more-than-half-of-earths-heat/ It turns out that nuclear fission is Earth's main internal heat source.