Classical Electrons in a Magnetic Field

Final Project PHYS225

Fall 2014

The dynamics of a charged particle in a magnetic field is described by

$$m\frac{d\mathbf{v}}{dt} = q\mathbf{v} \times \mathbf{B} - \gamma \mathbf{v} \quad (1)$$

where m and q are the mass and charge of the particle respectively, $\mathbf{B}$ iso the magnetic field and γ represents some sort of friction.

Part 1

Consider the case where the magnetic field is uniform, $\mathbf{B} = B_0\hat{k}$, and $\mathbf{v} = v_0\hat{\imath}$ and the particle is a proton. If $\gamma = 0$ then the object undergoes uniform circular motion. What is the radius of curvature of the motion?

There is a natural time scale for the problem which is $|m/(qB_0)|$. Rewriting (1) in this time scale gives (in component form)

$$\frac{dv_x}{d\tau} = v_y \quad (2)$$

$$\frac{dv_y}{d\tau} = -v_x \quad (3)$$

where $dt = |m/(qB_0)|d\tau$. Using Euler's method (see project 5) numerically solve these equations for positions and velocities.

Does Euler's Method provide stable solutions for this problem? Experiment with different step-sizes. Explain your results.

Create a function to numerically solve these equations using Heun's Method (see lecture notes for Project 5). Is Heun's method stable for this problem? Explain. Compare numerical solutions using Heun's Method with those of Euler's Method.

Part 2

Keeping the same uniform field incorporate friction as in equation (1). There are two natural time scales for this problem, $|m/(qB_0)|$ and $|m/\gamma|$.

Show that

$$v_x = +v_0 \cos\left[\left(\frac{qB}{m}\right)t + \phi_0\right] \exp\left[-\left(\frac{\gamma}{m}\right)t\right]$$

$$v_y = -v_0 \sin\left[\left(\frac{qB}{m}\right)t + \phi_0\right] \exp\left[-\left(\frac{\gamma}{m}\right)t\right]$$

are solutions to (1).

Using the better numerical method found in the previous section numerically solve equation (1). Test for various values of the time step.

Compare your chosen method to solutions computed using MATLAB's ODE45 function.

Compare your results to the analytical solutions provided above.

Part 3

You are now in a position to apply your chosen algorithm to a more interesting problem. In addition to the uniform magnetic field add an electric field in the x-direction, $\mathbf{E} = E_x\hat{\imath}$. Re-write equation (1) to incorporate this change. Use your chosen method (not ODE45) to solve this new set of equations. Try to investigate the behaviour of the system in various physical regimes. You should also vary the time step to check whether the stability conforms to your expectations. Think about the physical system you are describing and whether your results are consistent with the behaviour you would expect.

Part 4

Finally, consider making the electric field explicitly time dependent

$$E_x = E_0 \cos \omega t$$

and investigate the behaviour of the system as a function of frequency.

Your report

In your report you should describe how you have chosen which algorithm is most stable for the problem (1) and also your choice in time step. In addition you should think about the physics of the problem and try to identify the physically interesting parameter combinations. In particular you should ask yourself whether there is a qualitative change of behaviour at some value of the parameters or whether there is a value at which some special behaviour might be observed. If you cannot identify the physically interesting values, then start by doing a broad sweep of the meaningful parameters to try to identify any interesting features. Once you have found the interesting parameter ranges you can look at them more closely. Your report should contain a representative selection of results and a discussion of the physics which the illustrate. Please include a listing of your programs in your report as well as submitting the programs to dropbox.