Project 4: Integration

Due Dates

Week 7, Monday at 4:59

Your submission should be a single zip file containing all the .py files containing your code as well as a Single typed PDF document with your test results and answers to the problems/questions. Word documents will not be accepted. Your zip file should be in the form <lastname>_project_< N >.zip where you replace <lastname> with your last name and <N> with the project number. Upload your zip file via the link provided on the project webpage found at http://urminsky.ca.

In your PDF, please include your python code and any figures you were asked to plot.

Documentation: *Make sure that your code is well enough commented so that it can be easily understood by almost anyone, including yourself, if you look at it again at the end of the course (when you've forgotten how you coded it.)*

Project Briefing

Do not use any pre-defined integration methods included math or numpy libraries.

Part 1

The speed, v, at time t of a body of mass m, initially at rest, falling under gravity and subject to air resistance proportional to the speed is given by,

$$v = \frac{gm}{C_d}(1 - e^{(-C_d/m)t})$$

where C_d is the drag coefficient and $g = 9.81$ m/s^2, is the acceleration due to gravity.

The velocity in this case is not constant and therefore, the distance, s, fallen after a time interval $t = t'$ has elapsed, is given by the integral

$$s = \int_0^{t'} dy = \int_0^{t'} v(t)dt.$$

Solve this integral **by hand** and evaluate it to find s for a skydiver of mass $m = 70$ kg after $t = 10$ s has elapsed. Assume that $C_d = 12.5$ kg/s. You will need the result accurate to many digits for use in Part 2 where you will use this result.

Part 2

Design, write and test a PYTHON function that uses the Trapezoidal Rule to numerically integrate an arbitrary mathematical function y=f(x) between limits $x = x_L$ and $x = x_H$.

The function should be named int_trap and should **return** the value of the integral computed by the trapezoid method and have the form

```
def int_trap(func,lower,upper,N):
    ## your code here ##
    ...
    ...
    return value
```

lower: lower limit of integration

upper: upper limit of integration

N: number of elements (strips) used in integration.

func: is an arbitrary predefined function of one variable of the form:

```
def f(x):
    ##this function takes in x where x can be a float or a np.array and returns x squared###
    return x**2
```

Look into using numpy arrays for variables.

Use the program to find the distance fallen by the skydiver after 10 s, with the same parameters as given above. Run the program for N = 5, 10, 20 and 100 and calculate the **true fractional error** in each case using the result of your analytical calculation as the true value. Supposing you wish to know the distance to three significant figures, what is the best choice of N? Asking for more than three significant figures in this particular case of the skydiver isn't too smart — Why?

Part 3

Modify and create a new function int_trap2 that automatically adjusts N in order to achieve a pre-defined tolerance (relative error). the function should have the form

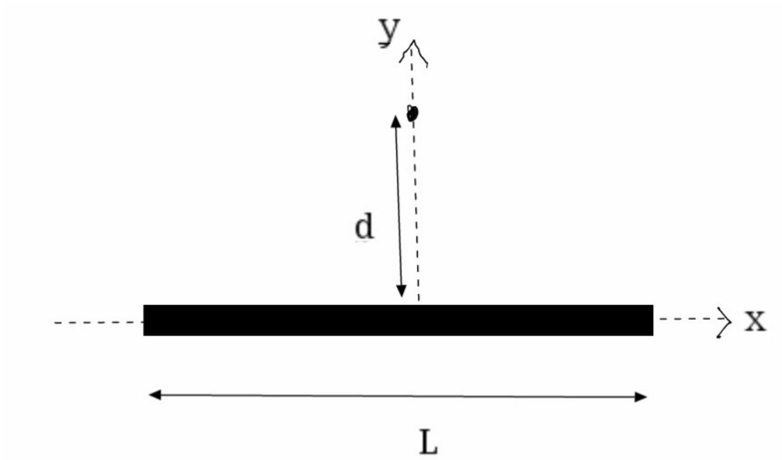
```
def int_trap2(func,lower,upper,N,TOL):
    ## your code here ##
    ...
    ...
    return value
```

where TOL is an arbitrary error bound and the other variables are as describe above.

Part 4 - Finding the electric field from a charge distribution

Create a new PYTHON function int_simp which computes the integral of a function of one variable using the Simpson Rule. Your function should have the form:

```
def int_simp(func,lower,upper,N):
    ## your code here ##
    ...
    ...
    return value
```



a. There is a charge distribution of $\lambda(x)$ over a rod of length L. If λ is an positive even function of x, show that the electric field, **E**, at a distance d on the perpendicular bisector is given by

$$\mathbf{E} = 2\int_0^{L/2} \frac{\lambda(x)dx}{4\pi\epsilon_0\left(x^2+d^2\right)^{\frac{3}{2}}}\hat{\jmath}$$

b. If $\lambda = 1.6 \times 10^-3$ C/m, find the magnitude of the above field using your version of int_simp where $d = 1.0$ m and $L = 1.0$ m. Compare this to the analytical value (you should be able to find the analytical value by hand).

c. If $\lambda = (1.23 \times 10^{-6}\text{C/m})(1 - \cos x)$, find the magnitude of the above field using int_simp to 4 significant figures.