Project 5: Solving ODE's - flight of a baseball

Due Dates

Week 8, Monday at 4:59

Your submission should be a single zip file containing all the .py files containing your code as well as a Single typed PDF document with your test results and answers to the problems/questions. Word documents will not be accepted. Your zip file should be in the form <lastname>_project_< N >.zip where you replace <lastname> with your last name and <N> with the project number. Upload your zip file via the link provided on the project webpage found at http://urminsky.ca.

In your PDF, please include your python code and any figures you were asked to plot.

No handwritten work (scanned or otherwise) will be accepted. Please include any mathematics in a typed format. You are welcome to use latex or word to do your write up.

Documentation: *Make sure that your code is well enough commented so that it can be easily understood by almost anyone, including yourself, if you look at it again at the end of the course (when you've forgotten how you coded it.)*

Project Briefing

Do not use any differential equation solving methods included math or numpy libraries.

Part 1

The goal of this week's project is to use Eulers method to explore a simple dynamics problem. Suppose that the baseball is hit vertically upwards. The ball has mass $m = 0.15$ kg and diameter $d = 0.075$ m and leaves the bat with a speed $v_0 = 50$ m/s. We are going to use a model for the drag force due to air resistance that is more realistic (at least at relatively large speeds where the air flow is turbulent) than we did in the previous projects. This model assumes that the magnitude of the drag force is proportional to the square of the speed (quadratic drag):

$$\boldsymbol{F}_d = -\frac{1}{2} C_d \rho A v \boldsymbol{v}$$

where C_d is the dimensionless drag coefficient, A is the cross-sectional area of the ball, and ρ is the density of air. Note that the somewhat strange way that we have written the dependence of the drag force on the velocity (as being proportional to negative the product of the speed v and the vector velocity $\mathbf{v}$ guarantees that it always acts in the direction **opposite** to the velocity with its magnitude being proportional to the square of the speed. For our purposes, we can assume that $C_d = 0.35$ and $\rho = 1.2$ kg/m^3.

Use Newton's 2nd law to write down the equations of motion of the baseball for the case where its motion is in the vertical direction. Take the upward direction as positive and be careful to preserve the drag as always acting in the opposite direction to the velocity. In the end, what you should have are two first-order differential equations of the form:

$$\frac{dv_y}{dt} = ... \quad \text{and} \quad \frac{dy}{dt} = ...$$

Now, on paper, solve these differential equations to get v_y and the vertical height y as functions of time **for the case when** $\rho = 0$. Hence, work out the maximum height reached by the ball and the total time of flight until it returns to the initial height, if it leaves the bat with an initial speed $v_0 = 50$ m/s in the y-direction, and using $g = 9.81$ m/s^2 for the acceleration due to gravity. Plot $v_y(t)$ and $y(t)$ versus time. You will need these results to test your PYTHON code.

Next, for the general case where air resistance is included, write down Euler equations for both the velocity v_y and the height y. Remember that these equations should relate the velocity and height at time t_{i+1} to the velocity and height at the previous time t_i. [You can suppress the y subscript on the velocity v in order to make the notation less cumbersome here, but just keep in mind that you are calculating the y-component of a vector and not just the magnitude of the vector.]

Part 2

Design, write, and test a PYTHON function that uses the Euler method to compute the trajectory of the baseball.

The program should

- Compute and print (to the screen) the maximum height reached by the ball.
- Compute and print (to the screen) the total time of flight of the ball.
- Write out to a file the values of time, velocity and height at each step in time.

The function should be named baseball and have the form

```
def baseball(t_step,height_0,vel_0,airFlag, outfile):
    #### your code here ####
    ...
```

The arguments are

t_step is the timestep Δt. That is $t_{i+1} = t_i + \Delta t$

height_0 initial height of ball

vel_0 initial velocity of the ball (in the y-direction)

airFlag Flag to determine whether or not to include air resistance

outfile the name of the out file to which you will write the values t, v and y at each time step.

You will also need to set the values of the constants g, C_d, m, and A within your program. Remember that the relevant values are $g = 9.81$ m/s^2, $C_d = 0.35$, and $m = 0.15$ kg. The balls cross-sectional (not surface!!!) area A is derived from the balls diameter $d = 0.075m$.
Use airFlag as a switch to turn air resistance on or off. This is mostly easily accomplished with the following trick: If airFlag = False then set $\rho = 0$. If airFlag = True then set $\rho = 1.2$ kg/m^3.

Test your program by running it without air resistance and comparing the results with your analytical solution for this case. Adjust the timestep so that your computed result for the maximum height matches the exact result to within 0.1 m.

When you are satisfied, use the function to computed the trajectory of the baseball for the following cases.

1. airFlag = False, vel_0 = 20 m/s
2. airFlag = False, vel_0 = 50 m/s
3. airFlag = True, vel_0 = 20 m/s
4. airFlag = True, vel_0 = 50 m/s

In all cases, assume height_0 = 0. Make a table giving the time of flight and maximum height in each case. Comment briefly on your results.

Use the data contained in the output files to make graphs plotting (a) v_y versus t; (b) y versus t. You will find a PYTHON function that reads and plots data from an input file on the course web page: plot_from_file.py. You are strongly encouraged to make the plots using PYTHON.

Writing data to a file

Your program will have a main loop, which updates the time and computes new values of v_y and y. The following lines will, respectively, open an output file, write the current value of time, height, and velocity to this file and then close the output file.

```
# Open the output file.
# This should be done before the main loop.
# the filename is a string variable storing the name of the file.
filename = 'file.txt'
f1 = open(filename)
....
# Write data to the file in the main loop
print >>f1, t, v_y, y
....
# At the end of your function you then close the file
f1.close()
```

You may use and adapt the files write_trig.py and plot_from_file.py to use with your project. These can be found on the project webpage. When you are designing your program you should consider the following:

- How will you determine the maximum height?
- When should you stop the integration?

Part 3

Create a new program so that it computes the trajectory of the baseball for the general case when the initial velocity of the ball as it leaves the bat is directed at an angle θ to the horizontal. You will need to compute both the vertical y and horizontal x positions and therefore the initial velocity should be resolved into vertical and horizontal components. (Hint: Gravity acts only in the vertical direction; The drag force acts in the direction directly opposite to the velocity and hence will affect both the x and y components.) The angle θ should be given as an input parameter. Also replace v_y in the output file with x. Use your program to compute trajectories for different values of the input parameters. For example, keep the magnitude of the initial velocity fixed and find the angle for which the ball flies the largest horizontal distance from the bat both with and without drag, Plot trajectories (i.e., height [y] versus horizontal position [x]) for cases of interest.