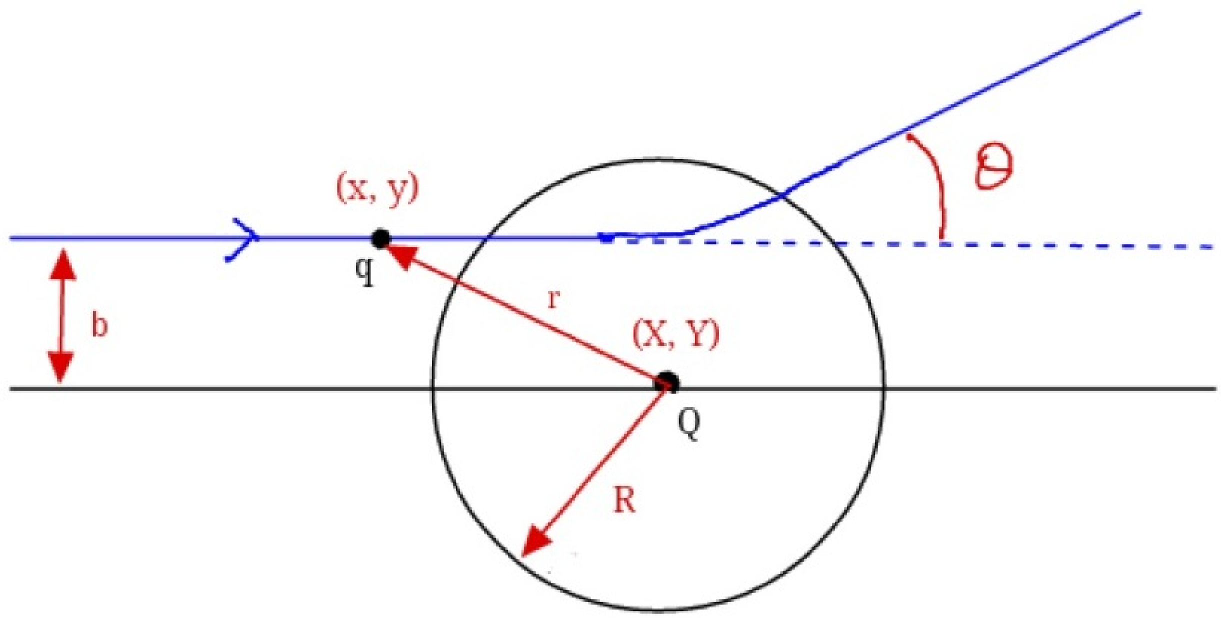
Rutherford Scattering

Final Project PHYS225

Fall 2014



This project deals with one of the most fundamental experiments in the last century or two which was first suggested by Lord Rutherford and carried out by Geiger and Marsden. At the time, the structure of atoms was still a complete mystery. The problem was size, you just cannot see atoms through a microscope. Rutherford proposed to study the distribution of mass and charge in the atom by shooting alpha particles at atoms and observing the results.

The physics of the problem is fairly straight forward. Find the electric field strength **E** in the vicinity of a sphere (the atom) or + charge, and also find **E** inside the sphere, assuming that the charge is uniformly distributed. The force, $\mathbf{F} = q\mathbf{E}$, on the alpha particle can then be calculated a it approaches the atom and/or enters the atom. One we have the force, we can then find the acceleration on the alpha particle. Also, from Newton's 3rd Law, the atom also accelerates and moves about.

The geometry of the problem is shown in the above figure. We idealise the atom as a sphere of radius R and to simply the problem we will assume it is a two dimensional problem. The figure illustrates one possible trajectory for an alpha particle. The nucleus, having positive charge, repels it, and it is bent away from a straight-line through a scattering angle θ. The quantity labelled b in the figure is called the impact parameter. It is the distance from an axis through the centre of the nucleus to the parallel trajectory of an incoming alpha particle. Assuming the atom has a net positive charge Q located at (X, Y) and has radius R, and the alpha particle is at (x, y) and has charge q. The vector $\mathbf{r}$ is from Q to q. Outside the atom $(r > R)$ the electric field is simply

$$\mathbf{E} = k\frac{Q}{r^2}\frac{\mathbf{r}}{r}$$

and inside the atom, $(r < R)$ the field is

$$\mathbf{E} = \frac{r^3}{R^3}k\frac{Q}{r^2}\frac{\mathbf{r}}{r}.$$

Part 1

Using the variables above write out the x and y components of the acceleration vector for both the alpha particle (a_x and a_y) and the atom (A_x and A_y). With m and M being the masses of the alpha particle and atom respectively. It is recommend you show your instructor your equations before you continue with the rest of this project.

Part 2

For this project you will need to numerically solve the equations you found in the previous section using the Leapfrog method. Notice that the acceleration equations have only one independent variable which is position. Also, your velocity equations have only one independent variable being velocity

$$\frac{dv}{dt} = f(\mathbf{r})$$
$$\frac{dx}{dt} = g(\mathbf{v}).$$

You can then write a numerical scheme like

$$\mathbf{v_{n+1/2}} = \mathbf{v_n} + \frac{h}{2}f(\mathbf{r_n})$$
$$\mathbf{r_{n+1}} = \mathbf{r_n} + hg(\mathbf{v_{n+1/2}})$$
$$\mathbf{v_{n+1}} = \mathbf{v_{n+1}} + \frac{h}{2}f(\mathbf{r_{n+1}})$$

where subscript represents the time step. Note that we first take a half time step, $h/2$, to calculate a half velocity. We then use the half velocity to compute the new position over a time step h. Finally you use the new position to calculate the new velocity using another half step.

Your program should take in initial velocity of alpha particle and atom, initial positions of each, time step, the radius of the atom and impact parameter as user provided variables and compute the trajectory of both the alpha particle and the atom.

Part 3

Try your program with the following parameters. Let the target atom be gold with positive charge $79 \times (1.60 \times 10^{-19})$ C, with radius 1×10^{-13}m. Try impact parameters larger and smaller than the radius. Assume that the kinetic energy of the alpha particle is 7 MeV. You can assume that initial position and velocity of the atom are $X = Y = 0$ and $V_x = V_y = 0$. Note that the time scale for this problem is very short. Experiment with different time steps. What time step works best for this problem.

Run the program with radius values 10^{-11}, 10^{-12}, 10^{-13}, 10^{-14}, and 2×10^{-15} meters. For each radius, you should run your experiment with 10 different values of the impact parameter ranging from approximately 10^{-13} to 10^{-14} m. You should produce a figure that shows how the scattering depends on the impact parameter for that particular radius of the atom.

Your Report

Your report should include explanations for your chosen time step for the algorithm as well as figures asked for in Part 3. After seeing the results, and knowing that, in fact, alpha particles are scattered backward, what would you conclude about the size of the target, which we may no properly call a nucleus? Explain why a target nucleus with radius of 10^{-11} meters does not scatter the alpha particle, but a nucleus with a radius of 10^{-14} meters is an effective scatterer.

From your results what can you infer about the structure of an atom? Use your results to justify your explanation.

Determine if your program conservers momentum and energy? Explain your results.

Finally, for a few examples, vary the force law so that the force is proportional to $1/r^4$ and repeat a few of the experiments. Describe how the results of this force law differ from a $1/r^2$ law.