#1

$$\vec{a} = \ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

$$m(\ddot{r} - r\dot{\theta}^2)\hat{r} + m(r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta} = F(r)\hat{r}$$

equating like terms yeilds

$$m(\ddot{r} - r\dot{\theta}^2) = F(r) \quad (1)$$

and

$$m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 0 \quad (2)$$

$* r$ from (2) $m(r^2\ddot{\theta} + 2r\dot{r}\dot{\theta}) = 0$

$$(*) \quad \frac{d}{dt}(mr^2\dot{\theta}) = 0 \quad //$$

From $(*)$ $\quad r^2\dot{\theta} = L \Rightarrow \dot{\theta} = \frac{L}{mr^2}$

So (1) Becomes

$$m\left(\ddot{r} - r\frac{L^2}{m^2r^4}\right) = F(r)$$

$$m\ddot{r} - \frac{L^2}{mr^3} = F(r)$$

c) $r(\theta) = Ae^{\beta\theta}$

$$L = m r^2 \dot{\theta}$$

$$d\theta = \frac{L}{mr^2} dt$$

$$r^2 d\theta = \frac{L}{m} dt$$

$$\int_{\theta_0}^{\theta} e^{2\beta\theta} d\theta = \int_0^t \frac{L}{m} dt$$

$$\frac{A^2}{2\beta} e^{2\beta\theta} \Big|_{\theta_0}^{\theta} = \frac{L}{m} t$$

$$t = \frac{A^2 m}{2L\beta}\left(e^{2\beta\theta} - e^{2\beta\theta_0}\right)$$

$$\theta(t) = \frac{1}{2\beta} \ln\left(\frac{2\beta L}{mA^2} t + e^{2\beta\theta_0}\right)$$

$$r = Ae^{\ln\left(\frac{2\beta L t}{mA^2} + e^{\beta\theta_0}\right)^{1/2}}$$

$$r = A\sqrt{\frac{2\beta L t}{mA^2} + \beta\theta_0}$$

$$r(\theta) = Ae^{\beta\theta}$$

$$r(t) = Ae^{\beta\left[\frac{1}{2\beta}\ln\left(\frac{2\beta L}{mA^2}t + e^{2\beta\theta_0}\right)\right]}$$

$$= Ae^{\ln\left[\left(\frac{2\beta L}{mA^2}t + e^{2\beta\theta_0}\right)^{1/2}\right]}$$

$$r(t) = A\sqrt{\frac{2\beta L}{mA^2}t + e^{2\beta\theta_0}}$$

2)

$$F = -\frac{dU}{dx}$$

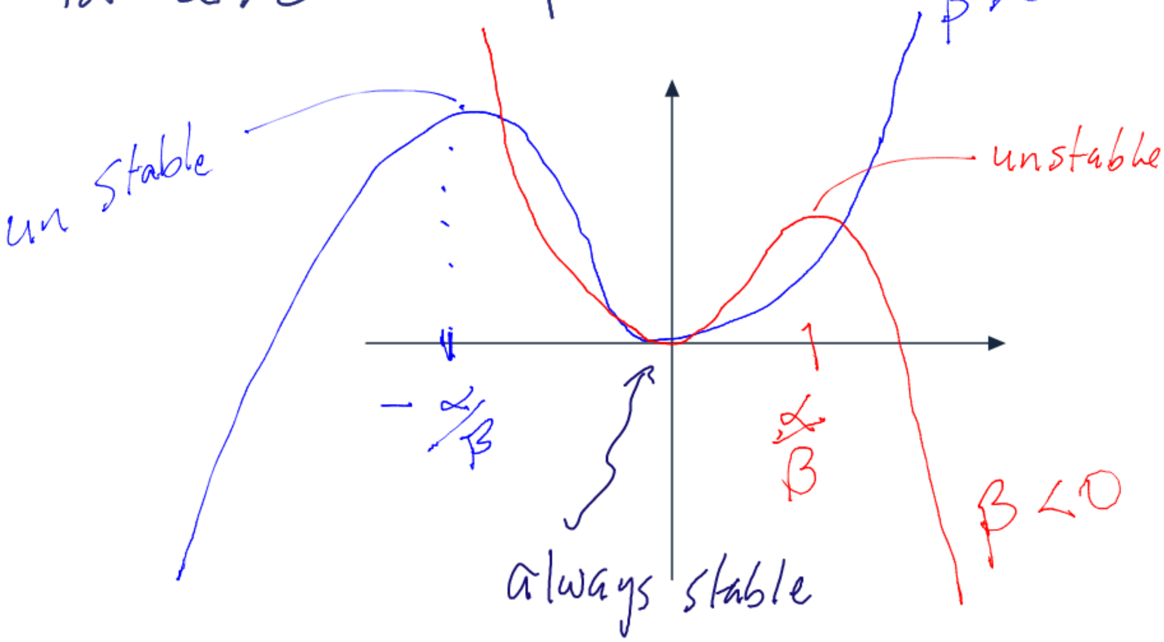
$$\frac{dU}{dx} = K_1 x + K_2 x^2 + K_3 x^3 + \dots$$

$$\Rightarrow F = m\ddot{x}$$

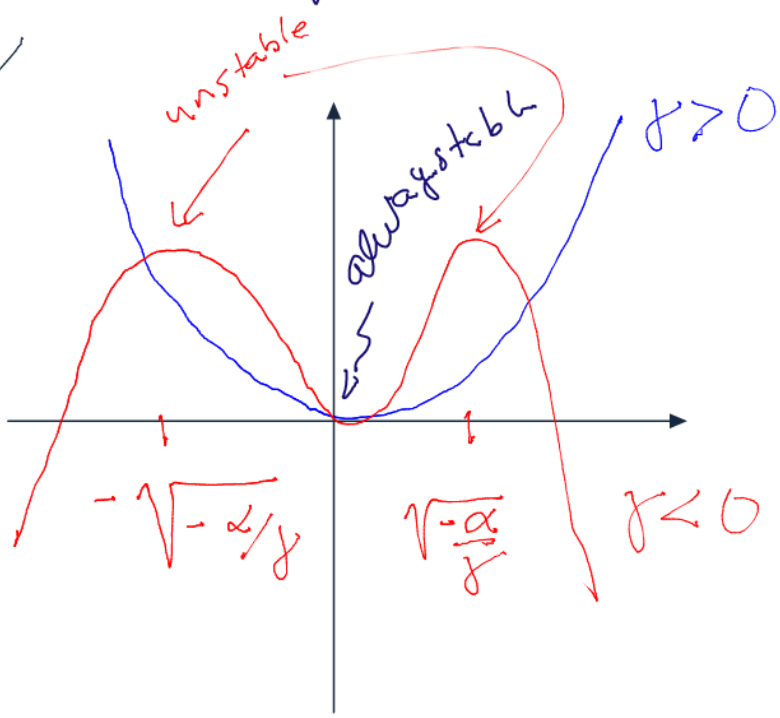
$$\ddot{x} + \alpha x + \beta x^2 + \gamma x^3 + \dots = 0$$

$$\alpha = \frac{K_1}{m} \quad \beta = \frac{K_2}{m} \quad \gamma = K_3/m$$

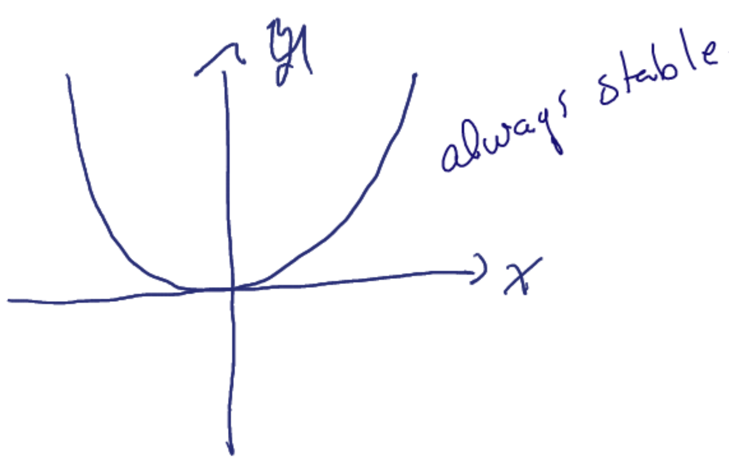
for $\alpha > 0$ and β

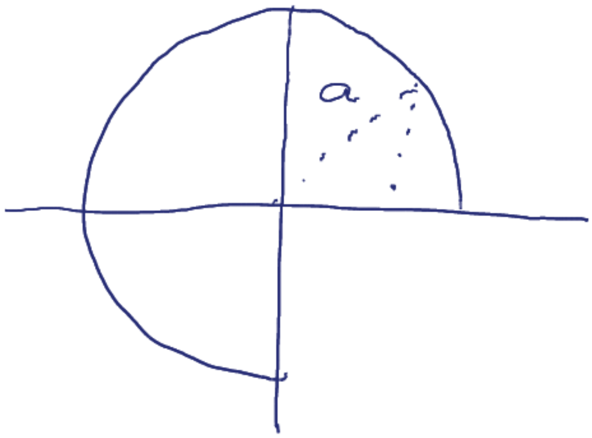


$\alpha > 0$ γ



for $\gamma > 0$





$$\rho = \frac{2M}{3\pi a}$$

$$M = \rho \frac{3\pi a}{2}$$

$$\bar{y} = \frac{1}{M}\int y\, dm$$

$$y = a\sin\theta$$
$$x = a\cos\theta$$
$$dm = \rho a\, d\theta$$

$$= \frac{1}{M}\int a\sin\theta\, \rho a\, d\theta$$

$$= \frac{a^2\rho}{M}\int_0^{3\pi/2} \sin\theta\, d\theta$$

$$= \frac{a^2\rho}{M}\left(-\cos\theta\Big|_0^{3\pi/2}\right) = \frac{a^2}{M}\rho\,(0+1)$$

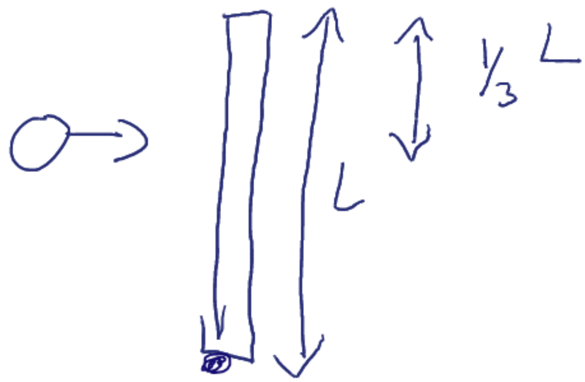
$$= \frac{a^2}{\rho\frac{3\pi}{2}a}\rho = \frac{2a}{3\pi}$$

$$\bar{x} = \frac{1}{M}\int a\cos\theta\, \rho a\, d\theta = \frac{a^2\rho}{M}\int_0^{3\pi/2} \cos\theta\, d\theta$$

$$= \frac{a^2\rho}{M}\left(\sin\theta\Big|_0^{3\pi/2}\right)$$

$$= -\frac{a^2\rho}{M} = -a\frac{2}{3\pi}$$

#4



$$m_b = \frac{1}{4} m_r$$

$$L_i = m_b v_0 \left(\frac{2L}{3}\right) = \frac{1}{4} m_r v_0 \frac{2}{3} L = \frac{1}{6} m_r v_0 L$$

$$L_f = \left(\frac{1}{3} m_r L^2 + m_b \left(\frac{2}{3} L\right)^2\right) \omega$$

$$= \left(\frac{1}{3} m_r L^2 + \frac{m_r}{4} \left(\frac{4}{9} L^2\right)\right) \omega$$

$$= \frac{4}{9} m_r L^2 \omega$$

$$L_i = L_f \Rightarrow \omega = \frac{3}{8} \frac{v_0}{L}$$

$$\frac{K_i}{K_f} = 4$$

#5

$$m\dot{v} = -f_f - cv^2$$

$$\int_{v_0}^{v} \frac{-m\,dv}{f_f + cv^2} = \int_0^t dt \qquad \text{sub } \frac{cv^2}{f_f} = u^2$$

$$u = \tan w$$

$$t = \frac{m}{\sqrt{f_f c}}\left(\tan^{-1}\left(\sqrt{\frac{c}{f_f}}\,v_0\right) - \tan^{-1}\left(\sqrt{\frac{c}{f_f}}\,v\right)\right)$$

$f_f = N \qquad m = 50\text{ kg} \qquad v_0 = 10.0\text{ m/s} \qquad c = 0.2$

$t = ? \qquad v = 5\text{ m/s}$